Area of Trapezoids

This is the first activity that students can really do on their own. The previous shapes of the parallelogram and the triangle gave students a mental template of how to discover area formula: relate the shape to a rectangle and then relate the base and height. They could have done the previous formulas on their own, but working through the development of the formulas with them can give more of a structure for further work that they can do more independently.

Since students are probably already familiar with the formula of the rectangle, parallelogram and triangle at some level, we did not do a discovery approach numerically first leading to the formula, but went straight to the formula. The same is true for this and future formulas. Some adolescents will need to work with the materials more, doing measuring and calculations before discovering the pattern and then formalizing it. Others will need fewer examples and can come to the formula much quicker because they have done the work with different shapes. For this lesson and for the future lessons, the presentation will include just the formalization of the formula and not include numeric experimentation. But of course this should happen if students have had no previous experience, or need more practice in developing patterns to discover a formula.

There are several ways to discover and interpret the formula for the area of a trapezoid. One main way will be presented here, along with a guided template that students could use independent of the teacher. The lesson presented here could also be used as a model for discussions with students after they have done the discovery activity independently.

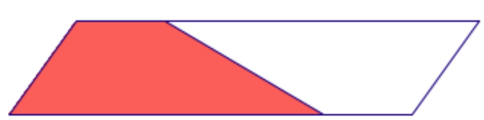
As always, these should be discussions with students after they have explored and developed ideas on their own. The dialogue here really should come from the students, or should be what we want students to say through our guided inquiry.

Presentation:

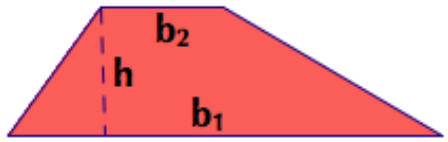
- Ask students to make a trapezoid, or we can give them trapezoids.

Method 1

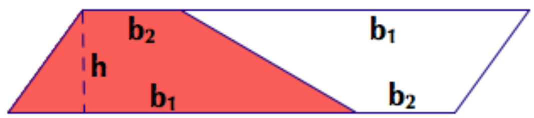
- SAY: "When we tried to find the area of a triangle, we could do it in many different ways. But one way which always worked was doubling the triangle and putting those two figures together to make a parallelogram. Try to do that for your trapezoid."
- Students can trace or cut our copies.
  - If we want, students can measure and calculate the area of the parallelogram, and thus the trapezoid for several examples. They can then see if they can discover a rule.



- SAY: "We know the trapezoid has area of half the parallelogram. But what are the base and the height of the new parallelogram in comparison with the original trapezoid? Well, the heights are both the same. We can draw that in. What about the base? If we look at our original trapezoid, it actually has two sides that can be used for the base that are different sizes. Let's call the long one b_1 and the other base b_2."



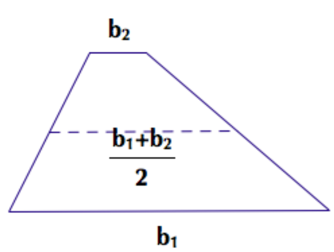
- SAY: "Now when we make our copy, we end up turning it upside-down in order to make our parallelogram. That means that the b_2 which is on top of the original, now is on the bottom and is our extension of our original b_1, making the base of the new parallelogram."



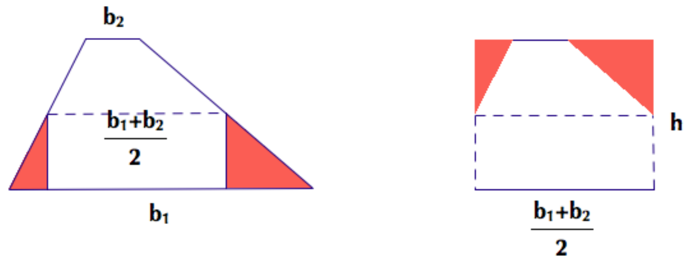
- SAY: "So how long is the base of the parallelogram? It is $b_1 + b_2$. Then we multiply this entire quantity by the height. So $(b_1 + b_2) \cdot h$. That is the area of the parallelogram in terms of the bases and height of the trapezoid. But what is the area of the trapezoid then? Yes, it is half this. We can write this as $A = \frac{(b_1 + b_2)h}{2}$. We can also write this in many different forms such as $A = \frac{1}{2}(b_1 + b_2) \cdot h$ and $A = \frac{b_1 + b_2}{2} \cdot h$."

Method 2

- SAY: "Here is another way to find the area. When we had a parallelogram and we decided what side was our base, the side opposite that side was the same size. So, it didn't matter which base we used. That is not the case for our trapezoid. Which base is the best to use? Some may say this one is too big, and perhaps this one is too small. We would like to use a base that is 'just right'. We can use a base that is right in the middle. We can find a number in between two numbers by taking their average. This means we add them together and divide by 2. Where would the average base be? Right between these two! If we connect the midpoints of the non-parallel sides, we will have a length that is the average of the two bases; $\frac{b_1 + b_2}{2}$."



- SAY: "Now we can make this into a rectangle by cutting the trapezoid perpendicular to our average base."



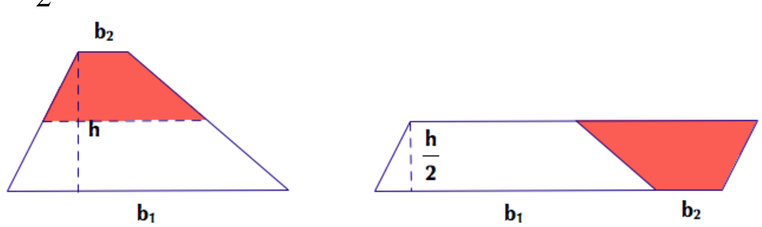
- SAY: "The area of our rectangle, which is the same as our trapezoid has for a base the average of the bases of the trapezoid, and the rectangle has the same height. We can see that the area of the trapezoid then is the *average* of the bases times the height, and we can write $A=\frac{b_1+b_2}{2}\cdot h$."

Follow-Ups:

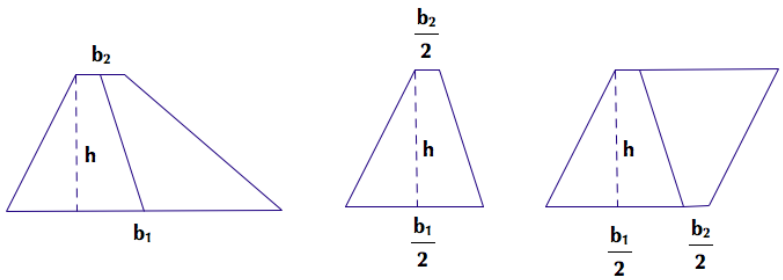
- Students can find the areas of trapezoids.
- Students can find unknown measurements of the trapezoid given the area.
- Students can give written explanations for the formula.
- Students can find alternate ways of justifying the area formulas (see Notes below).

Notes:

- The first way is the simplest for students to derive the formula and is most relate to the work of the triangle. The second way gives an easier way to remember the formula and is also similar to the work of the yellow material. There is an intuitive leap with the second way in the idea of relating the midpoints to the idea of the average, and that this midline will be parallel to the other bases.
- A third way to calculate the area is to divide the trapezoid into two smaller trapezoids along the midline. Then, make the parallelogram out of the two pieces, just like before. Here, the base will be b_1+b_2, but the height will be $\frac{1}{2}h$ or $\frac{h}{2}$. The area will then be $A=(b_1+b)\cdot\frac{h}{2}$.



- A fourth way would be to cut along the line connecting the midpoint of the bases. This actually creates two smaller trapezoids of equal area but different shapes. We can replicate either of the two halves to make a parallelogram with the same height as the original trapezoid. The new base or the parallelogram will be made or half of base 1 and half of base 2, giving $\frac{b_1}{2}+\frac{b_2}{2}=\frac{b_1+b_2}{2}$. This is a more unusual rearranging than the previous methods.



- Here is a guided activity which can allow students to discover the formula independently of a formal lesson.

There are many ways to find the area of a trapezoid. Here is one.
1) Make a trapezoid with construction paper so that the height is 3 inches, and the bases are 2 and 6 inches.
2) Cut this out.
3) Make an exact replica, and cut this out as well.
4) Now arrange these two trapezoids into a parallelogram.
5) Calculate the area of the parallelogram.
6) Use this information to deduce the area of the trapezoid.
Repeat this process for different trapezoids.
Write about how to find the area of a trapezoid, in terms of the bases and the height.
Create a formula for the area of a trapezoid, in terms of the bases and the height.
Make up some examples and solve them, or ask for examples to solve.
Can you find other ways to find the area?