FORMAL PRESENTATION OF THE PYTHAGOREAN THEOREM

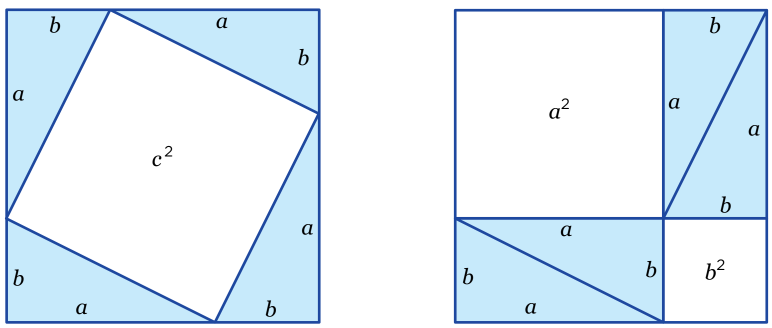
The students will have discovered the Pythagorean theorem through the previous activity, or perhaps they have known it before coming to our classroom. This lesson gives some history and vocabulary, as well as a formal proof relating the Pythagorean theorem to the Montessori material and Euclid's *Elements*. This lesson can be broken up or done at different times, depending on student interest and ability.

PREREQUISITES

Discovery of the Pythagorean Theorem

PRESENTATION

Revisit the previous discovery.



SAY "We have discovered that in any right triangle, $c^2 = a^2 + b^2$. When we have a right triangle, the side opposite the right angle is called the *hypotenuse*. The first part of this word comes from the Greek *hypo*- meaning *under*. Can you think of other words that have *hypo*- in them, and what the words mean?"

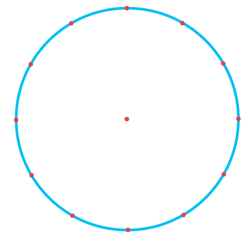
Examples may include *hypodermic* (under the skin), *hypothermia* (under temperature), and *hypochondria* (under the ribs).

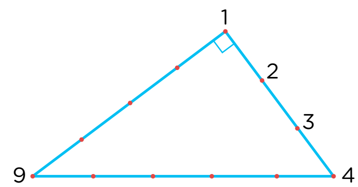
- *Hypo*- and *hyper*- are antonyms, meaning *under* and *over* respectively and are sometimes confused.

SAY "The second part of the word *hypotenuse* comes from the Greek *teinein*, meaning to *stretch*. What does this have to do with a triangle, to *stretch beneath*? Right angles were very important to the Greeks in things such as property and in making buildings. If you are in a field and you need to make a perfectly rectangular area, you need right angles. How can one measure right angles efficiently with the materials they had? People would make a rope that was 12 units long, with 12 knots equally spaced apart tied together in a circle. Then three people would each grab a knot. They would grab knots 1, 4, and 9, with knot 1 being at the right angle. The other two people holding knots 4 and 9 would form the right angle best they could. To make sure it was a right angle, they would *stretch* the rope beneath the right angle until it was tight. If they did this, it always made a right angle. Why would this be?"

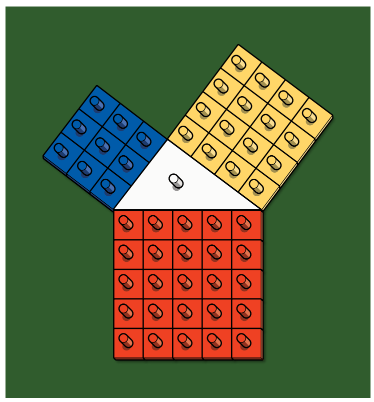
Students can discuss.

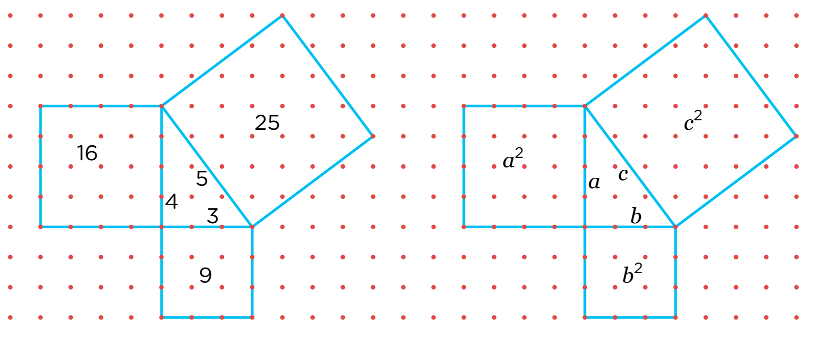
SAY "In our previous diagram, we can see in one picture a^2 and b^2, and in the other we see c^2. We also know that $c^2 = a^2 + b^2$ because they have the same area—we just rearranged the triangles to reveal this. However, we can also see this in just one diagram. Here is the Greek triangle. The sides correspond to $a = 3$, $b = 4$, and the hypotenuse, $c = 5$. By using this triangle and not rearranging anything, can we build these squares?"



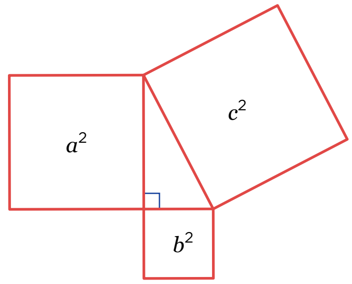


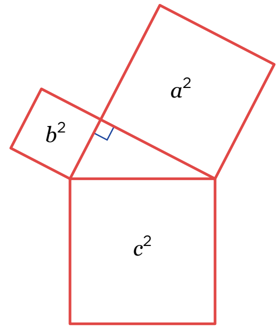
We can draw this or show the Montessori material.





SAY "This is the same triangle, just rotated so it looks like what we had before on our graph paper. We can see the hypotenuse, and the other two sides we call the *legs*. Instead of making a copy of the triangle, we can make the squares on the sides of our triangle. There is our c^2 on the hypotenuse, and there are our squares of a^2 and b^2 on the *legs* of our triangle. We can see that the squares on the legs add up to the square on the hypotenuse, since $9 + 16 = 25$. The Greeks and many civilizations before them knew that these three lengths made a right triangle exactly. Could other lengths be used? Yes, of course. In a right triangle, the squares on the legs together have area equal to that of the square on the hypotenuse. We can have the triangle in any position, like we did when we discovered this relationship, or like the Greeks might, with the hypotenuse 'stretched below.'"





SAY "This relationship is called the *Pythagorean theorem*. It is named after the Greek mathematician Pythagoras, who lived around 550 BCE. Did he invent the Pythagorean theorem? Certainly not! People knew about this for many centuries before Pythagoras, but it is named after him because he has been given credit for the proof that this relationship is true. How did he prove it? The same way that we did, by rearranging those triangles and showing that the areas $a^2 + b^2 = c^2$. The truth is, though, no one knows for sure if Pythagoras really did prove it this way, or if he was the first one, but people gave him credit for it and the name stuck. Pythagoras was influential and had many followers, so he is credited with many ideas and discoveries that may or may not have been exclusively his."

FOLLOW-UPS

- Students can revisit their original work with finding possible squares on dot paper with this new diagram.
- Students can solve for missing sides of a right triangle using the Pythagorean theorem.
- Students can write the Pythagorean proof for the theorem.
- Students can explore other ways of proving the relationship.
- Students can go on to other applications of the Pythagorean theorem.

NOTES

- If students want to construct the rope circle, they can do so as in the following diagram. They can try to make knots, which may be difficult due to the continual shortening of the rope (but it could be a fun challenge!) or they can simply mark the rope where the knots would be.