

The Math Institute Alignment with  
Mathematics Common Core Standards for  
Middle School



**The Math Institute**

*Mathematics | Middle School — Grade 6*

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
Sixth Grade – Ratios and Proportional Relationships – 6-RP			
Understand ratio concepts and use ratio reasoning to solve problems.			
TArA – Ch. 3 – <i>Ratio and Proportion</i>	6-RP 1		Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
TArA – Ch. 3 – <i>Calculating Unit Values</i>	6-RP 2	18A Unit Rates	Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” (Expectations for unit rates in this grade are limited to non-complex fractions.)
TArA – Ch. 3 – <i>Ratio and Proportion</i>	6-RP 3	5A Convert Percent to Fractions; 8B Convert Decimals to Percents; 11B Convert Percents to Decimals; 12C Convert Fractions to Percents; 15A Convert Decimals to Fractions	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

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TArA – Ch. 3 – <i>Ratio and Proportion</i>	6-RP 3a		Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
TArA – Ch. 3 – <i>Calculating Unit Values</i>	6-RP 3b		Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
TArA – Ch. 2 – <i>Converting Percents to Fractions and Decimals, Percent Questions</i>	6-RP 3c	25A Percentages	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
	6-RP 3d		Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
Sixth Grade – The Number System – 6-NS			
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.			
TArA – Chapter 1 - <i>Dividing Any Fraction by Any Fraction</i>	6-NS 1	14A Dividing Fractions	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$ . (In general, $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate

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			equally? How many $\frac{3}{4}$ -cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?
Compute fluently with multi-digit numbers and find common factors and multiples.			
	6-NS 2		Fluently divide multi-digit numbers using the standard algorithm.
	6-NS 3	4B Add / Subtract Decimals; 10B Multiplying Decimals; 15C Dividing Decimals	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
TArA – Chapter 3 - <i>Prime Factorization</i>	6-NS 4		Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$ .
Apply and extend previous understandings of numbers to the system of rational numbers.			
TAAV1 – Chapter 1 - <i>Introduction and Zero Sum Game</i>	6-NS 5		Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

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	6-NS 6	12B Graphing Points	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
	6-NS 6a		Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$ , and that 0 is its own opposite.
TAAV1 – Chapter 2 - <i>The Coordinate Plane</i>	6-NS 6b		Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
TAAV1 – Chapter 2 - <i>The Coordinate Plane</i>	6-NS 6c		Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6-NS 7	3C Absolute Value; 44A Writing Inequalities and Absolute Values	Understand ordering and absolute value of rational numbers.
	6-NS 7a		Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a

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			number line oriented from left to right.
	6-NS 7b		Write, interpret, and explain statements of order for rational numbers in realworld contexts. For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that $-3^{\circ}\text{C}$ is warmer than $-7^{\circ}\text{C}$ .
TAAV1 – Chapter 9 - <i>Absolute Value: Definition and Evaluation</i>	6-NS 7c		Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of $-30$ dollars, write $ -30  = 30$ to describe the size of the debt in dollars.
TAAV1 – Chapter 9 - <i>Absolute Value: Definition and Evaluation</i>	6-NS 7d		Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than $-30$ dollars represents a debt greater than 30 dollars.
TAAV1 – Chapter 2 - <i>Distance between Points Visually</i>	6-NS 8		Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Sixth Grade – Expressions and Equations – 6-EE			
Apply and extend previous understandings of arithmetic to algebraic expressions.			
TAAV1 – Chapter 6 - <i>Exponents as Repeated Multiplication</i>	6-EE 1	11A Evaluating Powers	Write and evaluate numerical expressions involving whole-number exponents.
	6-EE 2	16B Algebraic Vocabulary;	Write, read, and evaluate expressions in which letters stand for numbers.

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		19B Order of Operations	
TArA – Chapter 3 - <i>When to use an Operation</i>	6-EE 2a		Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract $y$ from 5” as $5 - y$ .
TArA – Chapter 3 - <i>When to use an Operation</i>	6-EE 2b		Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.
TAAV1 – Chapter 4 - <i>Evaluating Expressions by Using the Order of Operations</i>	6-EE 2c	23B Evaluating Expressions	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$ .
TAAV1 – Chapter 8 - <i>Distributive Property with Variables</i> , Chapter 9 - <i>Combining Like Terms in Abstraction</i>	6-EE 3	22C Distributive Property w/ Constants	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$ ; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$ ; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$ .

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TAAV1 – Chapter 9 - <i>Combining Like Terms in Abstraction</i>	6-EE 4	19C Combining Like Terms	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number $y$ stands for.
Reason about and solve one-variable equations and inequalities.			
TAAV1 – Chapter 5 - <i>Three Forms of Inequality Notation, Inequalities and the Number Line</i>	6-EE 5		Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
	6-EE 6		Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
TAAV1 – Chapter 4 - <i>Solving One-Step Equations with Positive Numbers</i>	6-EE 7		Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$ , $q$ and $x$ are all nonnegative rational numbers.
TAAV1 – Chapter 5 - <i>Three Forms of Inequality Notation, Inequalities and the Number Line</i>	6-EE 8	32A Inequality Notations	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Represent and analyze quantitative relationships between dependent and independent variables.			

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	6-EE 9		Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
Sixth Grade – Geometry – 6-G			
Solve real-world and mathematical problems involving area, surface area, and volume.			
TGA – Chapter 7 - <i>The Yellow Material, Area of Rectangles and Squares, Area of Parallelograms, Area of Triangles, Area of Trapezoids, Area of Kites and Rhombi, Area of Regular Polygons</i>	6-G 1	5C Areas of Rectangles and Squares; 8A Areas of Parallelograms; 10C Areas of Triangles; 41B Areas of Trapezoids; 45B Area of Rhombus and Kite; 48A Area of Polygons	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
TGA – Chapter 8 - <i>Volume of Right Prisms</i>	6-G 2		Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the

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			prism. Apply the formulas $V = l w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
TAAV1 – Chapter 2 - <i>The Coordinate Plane, Distance between Points Visually</i>	6-G 3		Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
TGA – Chapter 8 - <i>Building Solids</i>	6-G 4		Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Sixth Grade – Statistics and Probability – 6-SP			
Develop understanding of statistical variability.			
TArA – Chapter 6 - <i>Using Different Types of Graphs</i>	6-SP 1		Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
TArA – Chapter 6 - <i>Normal Distribution (Without Calculators)</i>	6-SP 2		Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

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TArA – Chapter 6 - <i>When to use a Measure of Central Tendency, Range, Variance &amp; Standard Deviation</i>	6-SP 3	2C Mode and Range; 6C Mean	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Summarize and describe distributions.			
TArA – Chapter 6 - <i>Using Different Types of Graphs, Box and Whisker Plots</i>	6-SP 4	13A Median and Quartiles; 16C Box and Whiskers Plots	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
	6-SP 5		Summarize numerical data sets in relation to their context, such as by:
	6-SP 5a		Reporting the number of observations.
	6-SP 5b		Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
TArA – Chapter 6 - <i>Central Tendency (Mean), Median, Mode, Range, Box and Whisker Plots</i>	6-SP 5c		Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
TArA – Chapter 6 - <i>When to use a Measure of Central Tendency</i>	6-SP 5d		Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

*Mathematics | Middle School — Grade 7*

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Seventh Grade – Ratios and Proportional Relationships – 7-RP			
Analyze proportional relationships and use them to solve real-world and mathematical problems.			
TArA – Chapter 3 - <i>Calculating Unit Values</i>	7-RP 1	15B Proportions / Ratios	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.
TArA – Chapter 3 - <i>Ratio and Proportion</i>	7-RP 2	15B Proportions / Ratios	Recognize and represent proportional relationships between quantities.
TArA – Chapter 3 - <i>Ratio and Proportion</i>	7-RP 2a		Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
TArA – Chapter 3 - <i>Ratio and Proportion</i>	7-RP 2b		Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
	7-RP 2c		Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$ , the relationship between the total cost and the number of items can be expressed as $t = pn$ .
	7-RP 2d		Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

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TArA – Chapter 2 - <i>Markups and Discounts, Simple Interest</i>	7-RP 3	17A Percentage Change; 21A Percent Proportions; 26A Simple Interest	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Seventh Grade – The Number System – 7-NS			
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.			
TArA – Chapter 1 - <i>Adding and Subtracting Fractions with Like Denominators, Adding and Subtracting Fractions with Unlike Denominators</i>	7-NS 1		Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
TAAV1 – Chapter 1 - <i>Introduction and Zero Sum Game</i>	7-NS 1a	1A Zero Sums / Opposites	Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
	7-NS 1b		Understand $p + q$ as the number located a distance $ q $ from $p$ , in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
TAAV1 – Chapter 4 - <i>Solving One-Step Equations with Positive Numbers, Solving One-Step Equations with Negative Numbers</i>	7-NS 1c		Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference,

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			and apply this principle in real-world contexts.
	7-NS 1d	3A Adding Like Integers; 5B Adding Unlike Integers; 7B Subtracting Integers; 8C Rewriting Integer Subtraction as Addition	Apply properties of operations as strategies to add and subtract rational numbers.
TArA – Chapter 1 - <i>Multiplying a Fraction by a Fraction, Dividing Fractions</i>	7-NS 2	12A Multiplying Integers	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
	7-NS 2a		Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
TAAV1 – Chapter 1 - <i>Dividing Signed Numbers</i>	7-NS 2b	13B Dividing Integers	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world contexts.

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TArA – Chapter 1 - <i>Multiplying a Fraction by a Fraction, Dividing Fractions</i>	7-NS 2c		Apply properties of operations as strategies to multiply and divide rational numbers.
TArA – Chapter 2 - <i>Converting between Decimals and Fractions</i>	7-NS 2d		Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
	7-NS 3		Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)
Seventh Grade – Expressions and Equations – 7-EE			
Use properties of operations to generate equivalent expressions.			
	7-EE 1	**31B Removing a common Factor	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
TArA – Chapter 2 - <i>Markups and Discounts, Simple Interest</i>	7-EE 2		Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.			
	7-EE 3		Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate

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			with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
	7-EE 4		Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
TAAV1 – Chapter 3 - <i>The Babysitting Problem</i> , Chapter 4 - <i>Solving Two-Step Equations</i>	7-EE 4a	3B Perimeter	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ , where $p$ , $q$ , and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
TAAV1 – Chapter 4 - <i>Solving Two-Step Equations</i>	7-EE 4b	7A Graphing Inequalities	<i>Solve word problems leading to inequalities of the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the</i>

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			<i>problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i>
Seventh Grade – Geometry – 7-G			
Draw, construct, and describe geometrical figures and describe the relationships between them.			
TGA – Chapter 9 – <i>Similarity, Ratios of Area and Volume, Similarity Projects</i>	7-G 1		Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
TGA – Chapter 4 - <i>Triangle Congruency Theorems</i>	7-G 2		Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
	7-G 3		Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.			
TGA – Chapter 6 - <i>Circumference of a Circle</i> , Chapter 7 - <i>Area of Circles</i>	7-G 4	11C Circumference; 14B Area of Circles	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

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TGA – Chapter 3 - <i>Adjacent Angles, Complementary and Supplementary Angles, Vertical Angles</i>	7-G 5	29B Supplementary / Complementary Angles; 31C Vertical Angles	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
	7-G 6	30A Surface Area	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Seventh Grade – Statistics and Probability – 7-SP			
Use random sampling to draw inferences about a population.			
	7-SP 1		Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
	7-SP 2		Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
Draw informal comparative inferences about two populations.			

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	7-SP 3		Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
TArA – Chapter 6 - <i>Central Tendency – Mean, Median, Mode, When to use a measure of Central Tendency</i>	7-SP 4		Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
Investigate chance processes and develop, use, and evaluate probability models.			
TArA – Chapter 4 - <i>Basic Probabilities</i>	7-SP 5		Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
TArA – Chapter 4 - <i>Basic Probabilities, The Monte Carlo Method</i>	7-SP 6		Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative

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			frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.the number of observations.
	7-SP 7	17C Basic Probability	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
TArA – Chapter 4 - <i>Basic Probabilities, Probability With and Without Replacement</i>	7-SP 7a		Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
	7-SP 7b		Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
TArA – Chapter 4 - <i>Compound Probability and Tree Diagrams, Probability with ‘or’ Statements</i>	7-SP 8	32B Simple Compound Probability	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

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TArA – Chapter 4 - <i>Compound Probability and Tree Diagrams</i>	7-SP 8a		Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
TArA – Chapter 4 - <i>Compound Probability and Tree Diagrams</i>	7-SP 8b		Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
	7-SP 8c		Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

*Mathematics | Middle School — Grade 8*

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
Eighth Grade – Ratios and Proportional Relationships – 8-RP			
Know that there are numbers that are not rational, and approximate them by rational numbers.			
TArA – Chapter 3 - <i>Classification of Numbers</i> , Chapter 2 - <i>Converting between Decimals and Fractions</i>	8-RP 1	14C Identify Irrational Numbers	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
	8-RP 2		Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$ ). For example, by truncating the decimal expansion of $\sqrt{2}$ , show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
Eighth Grade – Expressions and Equations – 8-EE			
Work with radicals and integer exponents.			
TAAV1 – Chapter 6 - <i>Multiplying Exponential Expressions, Dividing Exponential Expressions, Non-Positive Integer Exponents</i>	8-EE 1	21B Evaluating Negative Exponents; 25B Exponent Laws: Multiplication; 41C Exponent Laws: Multiplication; 43C Exponent Laws: Powers; 47C Exponent Laws: Zero/Negatives	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
TAAV1 – Chapter 4 - <i>Solving Simple Quadratic Equations, Solving Simple Equations of Any Power</i>	8-EE 2	1C Evaluating Square Roots; 4C Estimating Square Roots; 18B Evaluating Roots; 24A Solve Equations with Powers; 34C Complex Higher Powers	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ , where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
TArA – Chapter 3 - <i>Scientific Notation</i>	8-EE 3		Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$ , and determine that the world population is more than 20 times larger.
TArA – Chapter 3 - <i>Scientific Notation</i>	8-EE 4	24C Scientific Notation	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Understand the connections between proportional relationships, lines, and linear equations.			
TAAV1 – Chapter 3 - <i>The Babysitting Problem, Graphing Lines in Slope–Intercept Form</i>	8-EE 5		Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
			graph to a distance-time equation to determine which of two moving objects has greater speed.
TAAV1 – Chapter 3 - <i>The Babysitting Problem, Graphing Lines in Slope-Intercept Form, Equations of Parallel and Perpendicular Lines</i>	8-EE 6	24B Slope Given Graph; 31A Slope-Intercept Form; 36C Vertical and Horizontal Lines; 34B Parallel and Perpendicular Lines	Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$ .
Analyze and solve linear equations and pairs of simultaneous linear equations.			
TAAV1 – Chapter 4 - <i>Solving Linear Equations</i>	8-EE 7	20B Linear One Step w/ Add, Subt.; 22A Linear One Step w/ Mult., Div.; 38A Linear Equations with Fractions	Solve linear equations in one variable.
	8-EE 7a		Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$ , $a = a$ , or $a = b$ results (where $a$ and $b$ are different numbers).
TAAV1 – Chapter 4 - <i>Solving One-Step Equations with Fractional Multiples of <math>x</math>, Solving Multi-Step Equations</i>	8-EE 7b	29C Solving Complex Linear Equations	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
TAAV1 – Chapter 3 - <i>Substitution Method</i>	8-EE 8	30B Solving Systems via Substitution; 47A Systems of Equations by Elimination	Analyze and solve pairs of simultaneous linear equations.
TAAV1 – Chapter 3 - <i>Intersection of Lines: Visual Guess and Verification</i>	8-EE 8a		Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
TAAV1 – Chapter 3 - <i>Substitution Method, Elimination Method</i>	8-EE 8b		Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
	8-EE 8c		Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
<b>Eighth Grade – Functions – 8-F</b>			
Define, evaluate, and compare functions.			
TAAV2 – Chapter 3 - <i>Function Machines</i>	8-F 1		Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)
	8-F 2		Compare properties of two functions each represented in a different way (algebraically, graphically,

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
			numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
TAAV1 – Chapter 3 - <i>Graphing Lines in Slope-Intercept Form</i>	8-F 3		Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
Use functions to model relationships between quantities.			
	8-F 4		Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
	8-F 5		Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Eighth Grade – Geometry – 8-G			

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
Understand congruence and similarity using physical models, transparencies, or geometry software.			
TGA – Chapter 11 - <i>Transformations</i>	8-G 1		Verify experimentally the properties of rotations, reflections, and translations:
	8-G 1a		Lines are taken to lines, and line segments to line segments of the same length.
	8-G 1b		Angles are taken to angles of the same measure.
	8-G 1c		Parallel lines are taken to parallel lines.
TGA – Chapter 11 - <i>Transformations</i>	8-G 2		Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
TGA – Chapter 11 - <i>Transformations</i>	8-G 3		Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
TGA – Chapter 11 - <i>Transformations</i>	8-G 4		Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
TGA – Chapter 3 - <i>Interior Angles of a Triangle, Exterior Angles of a Triangle Theorem, Transversals with Parallel Lines -</i>	8-G 5	34A Angle Relations; 37B Transversals; 43A Exterior Triangle	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
Chapter 4 - <i>Angle-Angle (AAA)</i>			copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
Understand and apply the Pythagorean Theorem.			
TGA – Chapter 6 - <i>Discovering the Pythagorean Theorem, Formal Proofs of the Pythagorean Theorem</i>	8-G 6	36B Pythagorean Theorem 3-D	Explain a proof of the Pythagorean Theorem and its converse.
TGA – Chapter 6 - <i>Applying the Pythagorean Theorem</i>	8-G 7	17B Pythagorean Theorem 1; 20A Pythagorean Theorem 2; 23C Perimeter w/ Pythagorean Theorem	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
TAAV1 – Chapter 2 - <i>Distance Formula (Abstraction)</i>	8-G 8	23A Distance Formula	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.			
TGA – Chapter 8 - <i>Volume of Right Prisms, Volume of Oblique Prisms, Volume of Pyramids, Volume of a Sphere</i>	8-G 9		Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Eighth Grade – Statistics and Probability – 8-SP			
Investigate patterns of association in bivariate data.			
TArA – Chapter 6 – <i>Scatterplots, Correlation</i>	8-SP 1		Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association,

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard
			linear association, and nonlinear association.
TArA – Chapter 6 – <i>Correlation, Line of Best Fit</i>	8-SP 2		Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
TAAV1 – Chapter 3 - <i>The Babysitting Problem</i>	8-SP 3	29A Linear Story Problems	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
TArA – Chapter 6 – <i>Correlation</i>	8-SP 4		Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Math Institute Lesson	Common Core Standard	Integrated Math Sets 1-3 Skills	Description of Standard

## *Mathematics Standards for High School*

*The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:*

*(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).*

*All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.*

## *Mathematics | High School—Modeling*

Modeling Standards Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

*Mathematics | High School—Number and Quantity*

Math Institute Lesson	Common Core Standard	Description of Standard
Number and Quantity – The Real Number System – N-RN		
Extend the properties of exponents to rational exponents		
TAAV1 – Chapter 6 - <i>Fractional Exponents</i>	N-RN 1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(1/3)^3$ to hold, so $(5^{1/3})^3$ must equal 5.
TAAV1 – Chapter 6 - <i>Converting Fractional Exponents to Radicals</i>	N-RN 2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Use properties of rational and irrational numbers.		
TAA – Chapter 3 - <i>Classification of Numbers</i>	N-RN 3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Number and Quantity – Quantities – N-Q		
Reason quantitatively and use units to solve problems		
	N-Q 1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	N-Q 2	Define appropriate quantities for the purpose of descriptive modeling.
	N-Q 3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Number and Quantity – The Complex Number System – N-CN		
Perform arithmetic operations with complex numbers.		
TAAV2 – Chapter 7 - <i>Imaginary Numbers, Complex Numbers</i>	N-CN 1	Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.

Math Institute Lesson	Common Core Standard	Description of Standard
TAAV2 – Chapter 7 - <i>Powers of i, Addition, Subtraction, and Multiplication of Complex Numbers</i>	N-CN 2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
TAAV2 – Chapter 7 - <i>Complex Conjugates, Division of Complex Numbers</i>	N-CN 3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane.		
TAAV2 – Chapter 7 - <i>Graphing Complex Numbers</i>	N-CN 4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
TAAV2 – Chapter 7 - <i>Graphing Complex Numbers</i>	N-CN 5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .
	N-CN 6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Use complex numbers in polynomial identities and equations.		
TAAV1 – Chapter 10 - <i>The Quadratic Formula</i>	N-CN 7	Solve quadratic equations with real coefficients that have complex solutions.
TAAV2 -Chapter 5 - <i>Factorization of Polynomials Over the Complex Numbers</i>	N-CN 8	(+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
TAAV2 -Chapter 5 - <i>The Fundamental Theorem of Algebra</i>	N-CN 9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Vector and Matrix Quantities - N-VM		
Represent and model with vector quantities.		
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed

Math Institute Lesson	Common Core Standard	Description of Standard
		line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v$ , $ v $ , $  v  $ , $v$ ).
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors.		
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 4	(+) Add and subtract vectors.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 4a	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 4c	Understand vector subtraction $v - w$ as $v + (-w)$ , where $-w$ is the additive inverse of $w$ , with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 5	(+) Multiply a vector by a scalar.
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 5a	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ .
TAAV2 – Chapter 8 - <i>Vectors</i>	N-VM 5b	Compute the magnitude of a scalar multiple $cv$ using $  cv   =  c v$ . Compute the direction of $cv$ knowing that when $ c v \neq 0$ , the direction of $cv$ is either along $v$ (for $c > 0$ ) or against $v$ (for $c < 0$ ).

Math Institute Lesson	Common Core Standard	Description of Standard
Perform operations on matrices and use matrices in applications		
TAAV2 – Chapter 8 - <i>Matrices</i>	N-VM 6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
TAAV2 – Chapter 8 - <i>Matrices</i>	N-VM 7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
TAAV2 – Chapter 8 - <i>Matrices</i>	N-VM 8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
TAAV2 – Chapter 8 - <i>Matrices</i>	N-VM 9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
TAAV2 – Chapter 8 - <i>Matrices</i>	N-VM 10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	N-VM 11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
	N-VM 12	(+) Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Math Institute Lesson	Common Core Standard	Description of Standard
Algebra – Seeing Structure in Expressions – A-SSE		
<i>Interpret the structure of expressions</i>		
	A-SSE 1	<i>Interpret expressions that represent a quantity in terms of its context. ★</i>
	A-SSE 1a	<i>Interpret parts of an expression, such as terms, factors, and coefficients.</i>
	A-SSE 1b	<i>Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>
TAAV1 – Chapter 8 - <i>Factoring Card Sorting</i>	A-SSE 2	<i>Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>
<i>Write expressions in equivalent forms to solve problems</i>		
	A-SSE 3	<i>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</i>
TAAV1 – Chapter 8 - <i>Complete Factorization</i>	A-SSE 3a	<i>Factor a quadratic expression to reveal the zeros of the function it defines.</i>
TAAV1 – Chapter 10 - <i>Finding the Vertex through Completing the Square</i>	A-SSE 3b	<i>Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</i>
	A-SSE 3c	<i>Use the properties of exponents to transform expressions for exponential functions. For example the expression <math>1.15^t</math> can be rewritten as <math>(1.151/12)^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>
TAAV2 – Chapter 2 - <i>The Sum of a Finite Geometric Sequence</i>	A-SSE 4	<i>Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. ★</i>
Algebra – Arithmetic with Polynomials and Rational Expressions - A -APR		
Perform arithmetic operations on polynomials.		

Math Institute Lesson	Common Core Standard	Description of Standard
	A-APR 1	<i>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</i>
<i>Understand the relationship between zeros and factors of polynomials</i>		
TAAV2 – Chapter 5 - <i>The Division Algorithm and Remainder Theorem</i>	A-APR 2	<i>Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</i>
TAAV2 – Chapter 5 - <i>Factorization of Polynomials Over the Rationals, Graphs of Polynomials</i>	A-APR 3	<i>Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</i>
<i>Use polynomial identities to solve problems</i>		
TAAV1 – Chapter 10 - <i>The Quadratic Formula</i> , TAAV2 – Chapter 7 - <i>Imaginary Numbers</i>	A-APR 4	Solve quadratic equations with real coefficients that have complex solutions.
TAAV2 – Chapter 5 - <i>Factorization of Polynomials Over the Complex Numbers</i>	A-APR 5	(+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
<i>Rewrite rational expressions.</i>		
TAAV2 – Chapter 5 - <i>Polynomial Division, The Division Algorithm and Remainder Theorem</i>	A-APR 6	<i>Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</i>
	A-APR 7	(+) <i>Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</i>

Math Institute Lesson	Common Core Standard	Description of Standard
<i>Algebra - Arithmetic with Polynomials and Rational Expressions - A-APR</i>		
<i>Perform arithmetic operations on polynomials</i>		
	A-APR 1	<i>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</i>
<i>Understand the relationship between zeros and factors of polynomials</i>		
<i>TAAV2 – Chapter 5 - The Division Algorithm and Remainder Theorem</i>	A-APR 2	<i>Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</i>
<i>TAAV2 – Chapter 5 - Factorization of Polynomials Over the Reals, Graphs of Polynomials</i>	A-APR 3	<i>Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</i>
<i>Use polynomial identities to solve problems</i>		
	A-APR 4	<i>Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity <math>(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</i>
<i>TAAV2 – Chapter 1 - Pascal's Triangle and Binomial Coefficients, Pascal's Triangle and the Binomial Theorem, Formalization of the Binomial Theorem</i>	A-APR 5	<i>(+) Know and apply the Binomial Theorem for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)</i>
<i>Rewrite rational expressions</i>		
<i>TAAV2 – Chapter 5 - Polynomial Division, The Division Algorithm and Remainder Theorem</i>	A-APR 6	<i>Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using</i>

Math Institute Lesson	Common Core Standard	Description of Standard
		<i>inspection, long division, or, for the more complicated examples, a computer algebra system.</i>
TAAV1 – Chapter 8 - <i>Multiplying and Dividing Rational Expressions, Adding and Subtracting Rational Expressions</i>	A-APR 7	<i>(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</i>
<b>Algebra - Creating Equations ★ - A-CED</b>		
<i>Create equations that describe numbers or relationships</i>		
	A-CED 1	<i>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>
	A-CED 2	<i>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</i>
TAAV1 – Chapter 5 - <i>Graphing Inequalities in Two Variables</i>	A-CED 3	<i>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>
TAAV1 – Chapter 4 - <i>Solving for Variables</i>	A-CED 4	<i>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i>
<b>Algebra - Reasoning with Equations and Inequalities - A-REI</b>		
<i>Understand solving equations as a process of reasoning and explain the reasoning</i>		
TAAV1 – Chapter 4 - <b><i>Solving Linear Equations</i></b>	A-REI 1	<i>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that</i>

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		the original equation has a solution. Construct a viable argument to justify a solution method.
TAAV1 – Chapter 4 - <i>Solving Simple Equations of Any Power</i>	A-REI 2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Solve equations and inequalities in one variable		
TAAV1 – Chapter 4 - <b><i>Solving Linear Equations</i></b> - Chapter 5 - <i>Solving Linear Inequalities</i> .	A-REI 3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
TAAV1 – Chapter 10 - <i>Solving Quadratic Equations by Factoring, The Quadratic Formula</i>	A-REI 4	Solve quadratic equations in one variable.
TAAV1 – Chapter 10 - <b><i>Completing the Square, The Quadratic Formula</i></b>	A-REI 4a	Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
TAAV1 – Chapter 4 - <i>Solving Simple Quadratic Equations</i> Chapter 10 - <i>The Quadratic Formula, Solving Quadratic Equations by Factoring</i>	A-REI 4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
Solve systems of equations		
TAAV1 – Chapter 3 - <i>Elimination Method</i>	A-REI 5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
TAAV1 – Chapter 3 - <i>Intersection of Lines: Visual Guess and Verification, Substitution Method, Elimination Method</i>	A-REI 6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

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TAAV1 – Chapter 3 - <i>Intersection of Lines: Visual Guess and Verification, Substitution Method</i>	A-REI 7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$ .
TAAV2 – Chapter 8 - <i>Matrices</i>	A-REI 8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
TAAV2 – Chapter 8 - <i>Matrices</i>	A-REI 9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
Represent and solve equations and inequalities graphically		
TAAV1 – Chapter 2 - <i>The Coordinate Plane</i>	A-REI 10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
	A-REI 11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
TAAV1 – Chapter 5 - <i>Solving Polynomial Inequalities from a Graph, Solving Polynomial Inequalities</i>	A-REI 12	Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

*Mathematics | High School—Functions*

Math Institute Lesson	Common Core Standard	Description of Standard
Functions – Interpreting Functions – F-IF		
<i>Understand the concept of a function and use function notation</i>		
TAAV2 – Chapter 3 - <i>Evaluating Using Function Notation, Definition of Function, Domain, and Range</i>	F-IF 1	<i>Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</i>
TAAV2 – Chapter 3 - <i>Evaluating Using Function Notation</i>	F-IF 2	<i>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</i>
	F-IF 3	<i>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i>
<i>Interpret functions that arise in applications in terms of the context</i>		
TAAV2 – Chapter 5 - <i>Graphs of Polynomials</i> - TAAV1 – Chapter 2 - <i>x- and y-Intercepts, Symmetry of Graphs</i>	F-IF 4	<i>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★</i>
TAAV2 – Chapter 3 - <i>Definition of Function, Domain, and Range</i>	F-IF 5	<i>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the</i>

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		<i>positive integers would be an appropriate domain for the function. ★</i>
	F-IF 6	<i>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★</i>
<i>Analyze functions using different representations</i>		
TAAV1 – Chapter 2 - <i>The Coordinate Plane</i>	F-IF 7	<i>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</i>
TAAV1 – Chapter 2 - <i>x- and y-Intercepts</i> , Chapter 10 - <i>Finding the Vertex from the x-Intercepts</i> , <i>Finding the Vertex through Completing the Square</i>	F-IF 7a	<i>Graph linear and quadratic functions and show intercepts, maxima, and minima.</i>
TAAV1 – Chapter 11 - <i>Parent Graphs</i>	F-IF 7b	<i>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</i>
TAAV2 – Chapter 5 - <i>Graphs of Polynomials</i> , <i>Factorization of Polynomials Over the Reals</i>	F-IF 7c	<i>Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</i>
TAAV2 – Chapter 5 - <i>Graphs of Rational Functions: Horizontal Asymptotes</i> , <i>Graphs of Rational Functions: Vertical Asymptotes</i> , <i>Complete Graphing of Rational Functions</i>	F-IF 7d	<i>(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</i>
TAAV2 – Chapter 4 - <i>Exponential Equations and Their Graphs</i> , <i>Logarithms as Functions</i> , Chapter 6 - <i>Graphing Sine and Cosine</i> , <i>Graphing Tangent</i> , <i>Graphing Reciprocal Functions</i>	F-IF 7e	<i>Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</i>
	F-IF 8	<i>Write a function defined by an expression in different but equivalent</i>

Math Institute Lesson	Common Core Standard	Description of Standard
		<i>forms to reveal and explain different properties of the function.</i>
TAAV1 – Chapter 10 - <i>Solving Quadratic Equations by Factoring, Finding the Vertex through Completing the Square</i>	F-IF 8a	<i>Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</i>
TAAV1 – Chapter 4 - <i>Formal Definition of Exponential, Equations Compound Interest</i>	F-IF 8b	<i>Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i>
	F-IF 9	<i>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>
<b>Functions – Building Functions – F-BF</b>		
<b>Build a function that models a relationship between two quantities</b>		
	F-BF 1	<i>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</i>
	F-BF 1a	<i>Determine an explicit expression, a recursive process, or steps for calculation from a context.</i>
	F-BF 1b	<i>Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i>

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TAAV2 – Chapter 3 - <i>Composition of Functions</i>	F-BF 1c	(+) <i>Compose functions. For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</i>
TAAV2 – Chapter 2 - <i>Generating a Formula for Any Term of an Arithmetic Sequence, Generating a Formula for Any Term of a Geometric Sequence</i>	F-BF 2	<i>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★</i>
<b>Build new functions from existing functions</b>		
TAAV1 – Chapter 11 - <i>Vertical Translations of Graphs, Horizontal Translations of Graphs, Transforming Graphs by a Scaling Factor</i> - Chapter 2 - <i>Symmetry with Function Notation</i>	F-BF 3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
TAAV2 – Chapter 3 - <i>Finding the Inverse of a Function</i>	F-BF 4	Find inverse functions.
	F-BF 4a	Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ .
TVAA2 – Chapter 3 - <i>Composition of Inverses</i>	F-BF 4b	(+) Verify by composition that one function is the inverse of another.
	F-BF 4c	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
	F-BF 4d	(+) Produce an invertible function from a non-invertible function by restricting the domain.
	F-BF 5	(+) Understand the inverse relationship between exponents and logarithms and

Math Institute Lesson	Common Core Standard	Description of Standard
		use this relationship to solve problems involving logarithms and exponents.
<b>Functions – Linear, Quadratic, and Exponential Models – F-LE</b>		
<i>Construct and compare linear, quadratic, and exponential models and solve problems</i>		
	F-LE 1	<i>Distinguish between situations that can be modeled with linear functions and with exponential functions.</i>
TAAV1 – Chapter 3 - <i>The Babysitting Problem</i> , TAAV1 – Chapter 4 - <i>Exponential Equations and Their Graphs</i>	F-LE 1a	<i>Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</i>
	F-LE 1b	<i>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</i>
TAAV1 – Chapter 4 - <i>Story Problems with Exponential and Logarithmic Equations</i>	F-LE 1c	<i>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</i>
	F-LE 2	<i>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</i>
	F-LE 3	<i>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</i>
TAAV2 – Chapter 4 - <i>Solving Simple Exponential and Logarithmic Equations</i>	F-LE 4	<i>For exponential models, express as a logarithm the solution to <math>ab^{ct} = d</math> where <math>a</math>, <math>c</math>, and <math>d</math> are numbers and the base <math>b</math> is 2, 10, or <math>e</math>; evaluate the logarithm using technology.</i>
<i>Interpret expressions for functions in terms of the situation they model</i>		
	F-LE 5	<i>Interpret the parameters in a linear or exponential function in terms of a context.</i>
<b>Functions – Trigonometric Functions – F-TF</b>		
<i>Extend the domain of trigonometric functions using the unit circle</i>		

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TAAV2 – Chapter 6 - <i>Radian Measure</i>	F-TF 1	<i>Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</i>
TAAV2 – Chapter 6 - <i>Radian Measure, Coterminal Angles</i>	F-TF 2	<i>Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</i>
TAAV2 – Chapter 6 - <i>Special Triangles, Discovering Coordinates of the Unit Circle</i>	F-TF 3	<i>(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosine, and tangent for <math>\pi-x</math>, <math>\pi+x</math>, and <math>2\pi-x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</i>
	F-TF 4	<i>(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</i>
<i>Model periodic phenomena with trigonometric functions</i>		
TAAV2 – Chapter 6 - <i>Graphing Sine and Cosine</i>	F-TF 5	<i>Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</i>
TAAV2 – Chapter 6 - <i>Graphing Sine and Cosine, Inverses with the Unit Circle</i>	F-TF 6	<i>(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</i>
TAAV2 – Chapter 6 - <i>Solving Trigonometric Equations</i>	F-TF 7	<i>(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★</i>
<i>Prove and apply trigonometric identities</i>		
TAAV2 – Chapter 6 - <i>Trigonometric Identities</i>	F-TF 8	<i>Prove the Pythagorean identity <math>\sin^2(\vartheta) + \cos^2(\vartheta) = 1</math> and use it to find <math>\sin(\vartheta)</math>, <math>\cos(\vartheta)</math>, or <math>\tan(\vartheta)</math> given <math>\sin(\vartheta)</math>, <math>\cos(\vartheta)</math>, or <math>\tan(\vartheta)</math> and the quadrant of the angle.</i>
TAAV2 – Chapter 6 - <i>Sum and Difference Formulas</i>	F-TF 9	<i>(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</i>

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*Mathematics | High School—Geometry*

Math Institute Lesson	Common Core Standard	Description of Standard
Geometry – Congruence – G-CO		
<i>Experiment with transformations in the plane</i>		
TGA – Chapter 2 - <i>Euclid’s Definitions</i>	G-CO 1	<i>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</i>
TGA – Chapter 11 - <i>Transformations</i>	G-CO 2	<i>Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</i>
TGA – Chapter 11 - <i>Transformations</i>	G-CO 3	<i>Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</i>
TGA – Chapter 11 - <i>Transformations</i>	G-CO 4	<i>Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</i>
TGA – Chapter 11 - <i>Transformations</i>	G-CO 5	<i>Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</i>
<i>Understand congruence in terms of rigid motions</i>		
TGA – Chapter 11 - <i>Transformations</i>	G-CO 6	<i>Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</i>

Math Institute Lesson	Common Core Standard	Description of Standard
TGA – Chapter 11 - <i>Transformations</i>	G-CO 7	<i>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</i>
TGA – Chapter 4 - <b><i>Triangle Congruency Theorems</i></b>	G-CO 8	<i>Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</i>
<b><i>Prove geometric theorems</i></b>		
TGA – Chapter 3 - <i>Vertical Angles, Transversals with Parallel Lines</i>	G-CO 9	<i>Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i>
TGA – Chapter 3 - <i>Interior Angles of a Triangle, Isosceles Triangle Theorem</i> , Chapter 5 - <i>Midlines</i>	G-CO 10	<i>Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i>
TGA – Chapter 5 - <i>Naming and Classification of Quadrilaterals, Parts of Quadrilaterals</i>	G-CO 11	<i>Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i>
<b><i>Make geometric constructions</i></b>		
TGA – Chapter 2 - <b><i>The Six Basic Constructions</i></b> ,	G-CO 12	<i>Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string,</i>

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<i>Construction Extensions and Activities</i>		<i>reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i>
TGA – Chapter 2 - <i>The Six Basic Constructions, Construction Extensions and Activities</i>	G-CO 13	<i>Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</i>
<b>Geometry – Similarity, Right Triangles, and Trigonometry – G-SRT</b>		
<i>Understand similarity in terms of similarity transformations</i>		
TGA – Chapter 11 - <i>Transformations.</i>	G-SRT 1	<i>Verify experimentally the properties of dilations given by a center and a scale factor:.</i>
	G-SRT 1a	<i>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</i>
	G-SRT 1b	<i>The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</i>
TGA – Chapter 9 - <i>Similarity</i>	G-SRT 2	<i>Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</i>
TGA – Chapter 4 - <i>Angle-Angle (AAA)</i>	G-SRT 3	<i>Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</i>
<b>Prove theorems involving similarity</b>		
TGA – Chapter 5 – <i>Midlines,</i> Chapter 6 - <i>Formal Proofs of the</i>	G-SRT 4	<i>Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two</i>

Math Institute Lesson	Common Core Standard	Description of Standard
<i>Pythagorean Theorem</i>		<i>proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i>
TGA – Chapter 4 - <i>Proofs Using Congruency Theorems</i>	G-SRT 5	<i>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</i>
<i>Define trigonometric ratios and solve problems involving right triangles</i>		
TGA – Chapter 9 - <i>Introduction of Similarity with Extensions to Trigonometry</i>	G-SRT 6	<i>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</i>
TAAV2 – Chapter 6 - <i>Sine and Cosine</i>	G-SRT 7	<i>Explain and use the relationship between the sine and cosine of complementary angles.</i>
TGA – Chapter 6 - <i>Applying the Pythagorean Theorem</i> Chapter 9 - <i>Introduction of Similarity with Extensions to Trigonometry</i>	G-SRT 8	<i>Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★</i>
<i>Apply trigonometry to general triangles</i>		
TAAV2 – Chapter 6 - <i>Law of Sines and Area of Triangles</i>	G-SRT 9	(+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
TAAV2 – Chapter 6 - <i>Law of Sines and Area of Triangles, Geometric Proof of the Law of Sines, Law of Cosines Algebraically</i>	G-SRT 10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
TAAV2 – Chapter 6 - <i>Law of Sines and Area of Triangles, Law of Cosines Algebraically</i>	G-SRT 11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
<b>Geometry – Circles – G-C</b>		
<b>Understand and apply theorems about circles</b>		
	G-C 1	Prove that all circles are similar.
TGA – Chapter 10 - <b><i>Angles of a Circle, Lines Through a Circle</i></b>	G-C 2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between

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		central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
TGA – Chapter 5 - <i>Centers of Triangles</i>	G-C 3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
TGA – Chapter 10 - <i>Tangents to Circles</i>	G-C 4	(+) Construct a tangent line from a point outside a given circle to the circle.
<i>Find arc lengths and areas of sectors of circles</i>		
TGA – Chapter 10 - <i>Central Angles and Arc Measure</i> , Chapter 6 - <i>Arc Length</i> – Chapter 7 - <i>Area of a Sector</i> TAAV2 – Chapter 6 – <i>Radian Measure</i>	G-C 5	<i>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</i>
<i>Geometry – Expressing Geometric Properties with Equations – G-GPE</i>		
<i>Translate between the geometric description and the equation for a conic section</i>		
TAAV2 – Chapter 8 - <i>Conic Sections</i>	G-GPE 1	<i>Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</i>
TAAV2 – Chapter 8 - <i>Conic Sections</i>	G-GPE 2	<i>Derive the equation of a parabola given a focus and directrix.</i>
TAAV2 – Chapter 8 - <i>Conic Sections</i>	G-GPE 3	<i>(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</i>
<i>Use coordinates to prove simple geometric theorems algebraically</i>		
	G-GPE 4	<i>Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i>

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TAAV1 – Chapter 3 - <i>Equations of Parallel and Perpendicular Lines, Combining Parameters</i>	G-GPE 5	<i>Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</i>
TGA – Chapter 2 - <i>Independent Construction Work For Students</i>	G-GPE 6	<i>Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</i>
TAAV1 – Chapter 2 - <i>Distance Formula (Abstraction)</i>	G-GPE 7	<i>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula . ★</i>
<b>Geometry – Geometric Measurement and Dimension – G-GMD</b>		
<i>Explain volume formulas and use them to solve problems</i>		
TGA – Chapter 6 - <i>Circumference of a Circle</i> , Chapter 7 - <i>Area of Circles</i> , Chapter 8 - <i>Volume of Right Prisms, Volume of Pyramids</i>	G-GMD 1	<i>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.</i>
TGA – Chapter 8 - <i>Volume of a Sphere</i>	G-GMD 2	<i>(+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</i>
TGA - Chapter 8 - <i>Volume of Right Prisms, Volume of Pyramids, Volume of a Sphere</i>	G-GMD 3	<i>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★</i>
<i>Visualize relationships between two-dimensional and three-dimensional objects</i>		
TGA – Chapter 8 - <i>Isometric Drawings, Building Boxes Activity</i>	G-GMD 4	<i>Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</i>
<b>Geometry – Modeling with Geometry – G-MG</b>		
<i>Apply geometric concepts in modeling situations</i>		
	G-MG 1	<i>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★</i>

Math Institute Lesson	Common Core Standard	Description of Standard
	G-MG 2	<i>(+) Understand that restricting a Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★</i>
	G-MG 3	<i>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★</i>

*Mathematics | High School—Statistics and Probability*

Math Institute Lesson	Common Core Standard	Description of Standard
Statistics and Probability – Interpreting Categorical and Quantitative Data SP-ID		
<i>Summarize, represent, and interpret data on a single count or measurement variable</i>		
TArA – Chapter 6 - <i>Box and Whisker Plots</i>	SP-ID 1	<i>Represent data with plots on the real number line (dot plots, histograms, and box plots).</i>
TArA – Chapter 6 - <i>Central Tendency: Mean, Median, Box and Whisker Plots, Variance &amp; Standard Deviation</i>	SP-ID 2	<i>Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</i>
	SP-ID 3	<i>Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</i>
TArA – Chapter 6 - <i>Normal Distribution (Without Calculators), Normal Distribution (With Calculators)</i>	SP-ID 4	<i>Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</i>
<i>Summarize, represent, and interpret data on two categorical and quantitative variables</i>		
	SP-ID 5	<i>Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</i>
TArA – Chapter 6 – <i>Scatterplots, Correlation</i>	SP-ID 6	<i>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</i>
TArA – Chapter 6 - <i>Line of Best Fit</i>	SP-ID 6a	<i>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i>

Math Institute Lesson	Common Core Standard	Description of Standard
	SP-ID 6b	<i>Informally assess the fit of a function by plotting and analyzing residuals.</i>
TArA – Chapter 6 - <i>Line of Best Fit</i>	SP-ID 6c	<i>Fit a linear function for a scatter plot that suggests a linear association.</i>
<i>Interpret linear models</i>		
TArA – Chapter 6 - <i>Line of Best Fit</i>	SP-ID 7	<i>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</i>
TArA – Chapter 6 – <i>Correlation, Correlation Coefficient</i>	SP-ID 8	<i>Compute (using technology) and interpret the correlation coefficient of a linear fit.</i>
TArA – Chapter 6 - <i>Correlation</i>	SP-ID 9	<i>Distinguish between correlation and causation.</i>
<i>Statistics and Probability – Making Inferences and Justifying Conclusions – SP-IC</i>		
<i>Understand and evaluate random processes underlying statistical experiments</i>		
	SP-IC 1	<i>Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</i>
???	SP-IC 2	<i>Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</i>
<i>Make inferences and justify conclusions from sample surveys, experiments, and observational studies</i>		
	SP-IC 3	<i>Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</i>
	SP-IC 4	<i>Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</i>
	SP-IC 5	<i>Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</i>
	SP-IC 6	<i>Evaluate reports based on data.</i>
<i>Statistics and Probability – Conditional Probability and the Rules of Probability – SP-CP</i>		

Math Institute Lesson	Common Core Standard	Description of Standard
Understand independence and conditional probability and use them to interpret data		
TArA – Chapter 4 - <i>Conditional Probability</i> , Chapter 7 - <i>Set Theory</i>	SP-CP 1	<i>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</i>
???	SP-CP 2	<i>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</i>
TArA – Chapter 4 - <i>Conditional Probability</i>	SP-CP 3	<i>Understand the conditional probability of A given B as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</i>
	SP-CP 4	<i>Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i>
TArA – Chapter 4 - <i>Conditional Probability</i>	SP-CP 5	<i>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the</i>

Math Institute Lesson	Common Core Standard	Description of Standard
		<i>chance of being a smoker if you have lung cancer.</i>
<i>Use the rules of probability to compute probabilities of compound events in a uniform probability model</i>		
TArA – Chapter 4 - <i>Conditional Probability</i>	SP-CP 6	<i>Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.</i>
	SP-CP 7	<i>Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model.</i>
TArA – Chapter 4 - <i>Bayes' Theorem</i>	SP-CP 8	<i>(+) Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model.</i>
TArA – Chapter 5 – <i>Permutations, Combinations</i>	SP-CP 9	<i>(+) Use permutations and combinations to compute probabilities of compound events and solve problems.</i>
<i>Statistics and Probability – Using Probability to Make Decisions – SP-MD</i>		
<i>Apply trigonometry to general triangles ??</i>		
	SP-MD 1	<i>(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.</i>
TArA – Chapter 4 - <i>Expected Value</i>	SP-MD 2	<i>(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.</i>
TArA – Chapter 4 - <i>Expected Value, Binomial Probability Distributions</i>	SP-MD 3	<i>(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four</i>

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		choices, and find the expected grade under various grading schemes.
TArA – Chapter 4 - <i>Expected Value, Binomial Probability Distributions</i>	SP-MD 4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
Use probability to evaluate outcomes of decisions		
TArA – Chapter 4 - <i>Expected Value</i>	SP-MD 5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
TArA – Chapter 4 - <i>Expected Value</i>	SP-MD 5a	Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
TArA – Chapter 4 - <i>Expected Value</i>	SP-MD 5b	Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
	SP-MD 6	(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
	SP-MD 7	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).